The Management University of Africa

Sponsored by the Kenya Institute of Management

UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 341: VECTOR ANALYSIS

DATE: 29TH JULY 2024

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provided.**

QUESTION ONE

(a) The following forces act on a particle P: F1 = 2 **i** + 3 **j** - 5 **k**, F2 = - 5 **i** + **j** + 3 **k**,

F3 = **i** – 2 **j** + 4 **k**, F4 = 4 **i** – 3 **j** – 2 **k**, measured in kgf. Find

(i) the resultant of the forces **(2 marks)**

(ii) the magnitude of the resultant **(2 marks)**

b) Find a vector parametrization for the line that passes through (1, 1, 1) and (3, - 5, 2) **(2 marks)**

c) Given **A** = - 3i + 2j + k and **B** = 8i - 6j + 2k, use the dot product obtain the cosine of the angle θ between **A** and **B** **(6 marks)**

d) Find the divergence of the vector field f = ($x^2y^2 + z^2$) **i** + x y z **j** + e^{xyz} **k** **(3 marks**

e) Find arc length line integral $\int_C f ds$ along the curve C given by y = 2 $\sqrt{x}$ from

x = 3 to x = 10 **(5 marks)**

f) Show that the integral $\int_C f.dr = \int_C (2x\,dx + 2y\,dy + 6z\,dz)$ is independent of the path in an domain in space and find its value if C has the initial point

p0 (0, 0, 0) and terminal point P (3, 3, 3) **(5 marks)**

QUESTION TWO

a) Find the dot product **A .B** given that **A** = 2 **i** - 3 **j** +7 **k** and

B = - 4**i** + 2 **j** - 4 **k** **(3 marks)**

b) If **A** = 3 **i** – **j** – 4 **k** , **B** = - 2 **i** + 4 **j** – 3 **k**, C = **i** + 2**j** – **k**, find:

i) 2A – B + 3 C **(4marks)**

ii) ||A + B + C|| **(3 marks)**

c) Determine a unit vector perpendicular l to the plane of

A = 2 **i** - 6 **j** - 3 **k** and **B** = 4**i** + 3 **j** - **k** . Hence calculate the sine of the angle between these vectors **(5 marks)**

QUESTION THREE

a) Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$

Find the component of its velocity in the direction **i** + 2 **j** – **k** at time t

(3 marks)

b) Compute the gradient of the function: $\emptyset(x, y, z) = 2x^3y + ze^y$ at point (1, 1, 2) and evaluate the maximum and minimum rate of change of the function at this point **(6 marks)**

c) Find the equation of the tangent plane and normal line $x^2 - 2y^2 + z^4 = 0$ the surface at the point (1, 1, 1) **(6 marks)**

QUESTION FOUR

a) Use Green's theorem to compute $\int_C \boldsymbol{F}.d\boldsymbol{R}$ assumes the curve is oriented counterclockwise **F** = (x + y) **i** + (x – y) **j**, C the ellipse $x^2 + 4z^2 = 1$ **(3 marks)**

b) Find the flux of the vector field $\mathbf{F} = xy^2\,\mathbf{i} + yz^2\,\mathbf{j} + zx^2\,\mathbf{k}$ across the surface S bounding the cylinder $2 \le x^2 + y^2 + z^2 \le 4,\ 0 \le z \le 6$ **(7 marks)**

c) Assuming Ω is a simply connected domain, test to see if F is conservative. If it is, find a potential function F = 2 x **i** – 2 y **j** + 2 z **k** **(5 marks)**

QUESTION FIVE

a) Find the area bounded by the parabola $y = x^2$ and the line y = 2x + 3

(4 marks)

b) Show that the integral $\int_C f.dr = \int_C (2x\,dx + 2y\,dy + 6z\,dz)$ is independent of the path in any domain in space and find its value if C has the initial point P0(0, 0, 0) and terminal point P(3, 3, 3) **(5 marks)**

c) Find the volume V of the solid S that is bounded by the elliptic paraboloid

$2x^2 + y^2 + z = 27$, the planes x = 3 and y = 3, and the three coordinate planes

(6 marks)

QUESTION SIX

a) If **A** = 3 **i** - **j** – 2 **k** and **B** = 2 **i** + 3 **j** + **k**, find **A** x **B** **(5 marks)**

b) Find the volume of a parallelopiped whose edges are OA, OB and OC, where

A (1, 2, 3), B (1, 1, 2) and C (2, 1, 1) **(5 marks)**

c) If **A** = a **i** - 2 **j** + **k** and **B** = 2 a **i** +a **j** - 4 **k**, are perpendicular vector s, find the value of a

(5 marks)