071506T4MTP

MECHANICAL TECHNOLOGY (PRODUCTION) LEVEL 6

ENG/OS/ME/CC/2/6/A

APPLY ENGINEERING MATHEMATICS

July/Aug. 2023

TVET CURRICULUM DEVELOPMENT, ASSESSMENT AND CERTIFICATION COUNCIL (TVET CDACC)

WRITTEN ASSESSMENT

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper has TWO sections: **A** and **B.**
2. You are provided with a separate answer booklet.
3. Marks for each question are as indicated.
4. Do not write on the question paper.
5. Use non-programmable scientific calculator.

This paper consists of FOUR (4) printed pages with FIFTEEN (15) questions

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing

Turn Over

SECTION A: (40 Marks)

*Answer **all** questions in this section*

1. Prove the trigonometric identity:

$$\frac{\tan\theta + \cot\theta}{\sec\theta \operatorname{cosec}\theta} = 1$$ (3 Marks)

2. Show that: $1 + 2sinh^2x = \cosh 2x$. (3 Marks)

3. If $\sin A = \frac{5}{13}$ and $\cos B = \frac{4}{5}$, where A and B are acute angles,

   determine: $\sin(A + B)$. (4 Marks)

4. Given that $y = x^2 \sin 2x$, determine: $\frac{dy}{dx}$ (4 Marks)

5. Given the complex numbers $z_1 = 4 + 5j$, $z_2 = 3 - 4j$ and $z_3 = 8 + 3j$:

   determine $z = z_3 + \frac{z_1 z_2}{(z_1 + z_2)}$ in the form $a + bj$ (4 Marks)

6. Solve the equation $3^{2x+2} + 3^{x+4} + 180 = 0$ (4 Marks)

7. Convert: $x^2 + y^2 = 7x$ to polar coordinates. (4 Marks)

8. The surface area of a sphere is given by $4\pi r^2$, where r is the radius. Determine the rate of change of the area in cm^2/second, when $r = 2\ cm$, given that the radius increases at the rate of 1 cm /second. (3 Marks)

9. Given the three vectors $\boldsymbol{A} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{B} = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\mathbf{C} = 2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$, determine the magnitude of the vector, $\boldsymbol{D} = \boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C}$

   (4 Marks)

10. Two cars P and Q are travelling towards the junction of two roads which are at right angles to one other. Car P has a velocity of 40 km/h due east and car Q a velocity of 50 km/h due south. Calculate the velocity of car P relative to car Q. (3 Marks)

11. Given $y = \frac{1}{x+3}$, determine $\frac{dy}{dx}$ from first principles. (4 Marks)

SECTION B: (60 Marks)

*Answer any **three** questions in this section.*

12. a) Use implicit differentiation to determine the equation of the;

    i. tangent,

    ii. normal,

    to the curve $9x^2 + 3y^2 - 4xy + 6x - 8y = 6$ at the point (1,3). (7 Marks)

    b) Locate the stationary points on the curve $z = 2x^2 + 3y^2 - 8xy + 6x - 6y$ and determine their nature. (7 Marks)

    c) Given that $z = \frac{x+y}{x-y}$, show that $x\frac{\partial^2 z}{\partial x^2} - y\frac{\partial^2 z}{\partial y^2} = 0$ (6 Marks)

13. a) The mean mass of a set of components is 740 kg and the standard deviation is 40kg. Assuming the masses are normally distributed about the mean, determine for a sample of 60 components, how many are likely to have masses of:

    i. less than 725 kg,

    ii. between 730kg and 750kg,

    iii. more than 750kg. (10 Marks)

    b) Table 1 below shows lengths of 200 drill bits in a pack.

    Table 1

Lengths (mm)	10-14	15-19	20-24	25-29	30-34	35-39	40-44
No. of bits	10	22	40	56	44	18	10

    Using an assumed mean of 27 mm, calculate the:

    i. mode

    ii. mean

    iii. standard deviation (10 Marks)

Turn Over

14. a) Three forces F_1, F_2 and F_3 in Newtons acting at a point in a mechanical system in Equilibrium satisfy the following three simultaneous equations:

$$F_1 - 2F_2 + F_3 = 0$$

$$-2F_1 + 3F_2 + 2F_3 = 17$$

$$3F_1 + 4F_2 - 3F_3 = 0$$

Use inverse matrix method to determine the value of the forces. (6 Marks)

b) i. Use Maclaurin's theorem to obtain the first four non-zero terms of the function: $f(x) = cos^2 x$,

ii. Hence, evaluate the integral $\int_0^1 \frac{cos^2 x}{x^{\frac{1}{3}}} dx.$ (8 Marks)

c) i. Express $4\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$ for $0^0 < \alpha < 90^0$

ii. Hence solve the equation $4\cos\theta - 3\sin\theta = 0.5$ for $0^0 \leq \theta \leq 360^0$. (6 Marks)

15. a) Find the general differential equation, given:

$$5\frac{dy}{dx} + 2x = 3,$$ (5 Marks)

b) i) Expand the following using binomial theorem,

$$(1 + 3x)^{1/2}$$

ii. Hence evaluate the value of $\sqrt{1.3}$. (6 Marks)

c) A scalar field ∅ and a vector field E**,** are given by:

E $= x^2y\boldsymbol{i} + xz^2\boldsymbol{j} + xy^2z\boldsymbol{k}$ and $\emptyset = x^2y^2z^2$. Determine at the point (1, 1, 1):

i. $|\boldsymbol{\nabla}\emptyset|$

ii. $\boldsymbol{\nabla}.\boldsymbol{E}$

iii. $\boldsymbol{\nabla} \times \boldsymbol{E}$ (9 Marks)

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