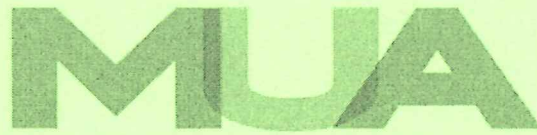


The
Management
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UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 323: CALCULUS III - APPLICATIONS

DATE: 1ST AUGUST 2024

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provide**

QUESTION ONE

- a) Verify Rolle's Theorem in the interval $(2, 4)$ for the function

$$f(x) = x^2 - 6x + 8 \quad (5 \text{ marks})$$

- b) Use the sum of the first 100 terms to approximate the sum of the series

$$\sum \frac{1}{(x^3+1)}. \text{ Estimate the error involved in this approximation.} \quad (5 \text{ marks})$$

- c) Form the partial differentiation equation by eliminating the constants a and b

$$\text{from the equation } (x + a)^2 + (y + b)^2 + z^2 = 4 \quad (5 \text{ marks})$$

- d) Evaluate $\int_0^3 \int_1^2 (x^2 + 3y^2) dy dx$ (5 marks)

- e) Evaluate $\int_C x^4 dx + x y dy$, where C is the triangular curve consisting of the line segments from $(0, 0)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$, and from $(0, 1)$ to $(0, 0)$

(5 marks)

QUESTION TWO

- a) Given $\int_0^3 (2x^2 - 1) dx = 15$, find C such that $f(c)$ equals the average value of $f(x) = 2x^2 - 1$ over $[0, 3]$ (3 marks)

- b) For the function $f(x) = x^2 + 2x$, find all the values of C that satisfy the Lagrange Theorem over the interval $(-4, 4)$ (4 marks)

- c) Determine if Rolle's Theorem applies for the function

$$f(x) = 3x - 3x^3 \text{ on } [-2, 1]. \text{ If it does, find } x = c \text{ guaranteed by it for } f(x) = 3x - 3x^3 \text{ on } [-2, 1] \quad (4 \text{ marks})$$

- d) Use L'Hospital's Rule to evaluate $\lim_{t \rightarrow 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$ (4 marks)

QUESTION THREE

- a) Use Taylor's Theorem to expand the function $f(x) = \sqrt[3]{x}$ in ascending powers of x up to degree 2. Approximate the function at $a = 8$ (6 marks)
- b) Use Maclaurin's theorem to expand $\ln(1+x)$ in ascending powers of x as far as the term $\ln x^5$ (6 marks)
- c) Find the sum $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ to within $\frac{1}{10}$ (3 marks)

QUESTION FOUR

- a) Use the chain to evaluate $y = (3x + 2)^5$ (3 marks)
- b) If $z = 2x^2 + 3xy + y^3$, find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ (4 marks)
- c) Find the second partial derivatives of
 a. $f(x, y) = x^3 + x^2y^3 - 2y^2$ (4 marks)
- d) A farmer has 100m of wire-mesh to fence a rectangular enclosure. What is the greatest area can he enclose with the wire-mesh? (4 marks)

QUESTION FIVE

- a) Evaluate the integrals $\int_0^3 \int_1^2 x^2 y \, dy \, dx$ (3 marks)
- b) Evaluate $\int_0^1 \int_0^1 \int_0^x (x - 2y + z) \, dz \, dy \, dx$ (6 marks)
- c) Evaluate $\int_C y \, dx + z \, dy + x \, dz$, where C consists of the line segment from C_1 from $(2, 0, 0)$ to $(3, 4, 5)$ followed by the vertical line segment and C_2 from $(3, 4, 5)$ to $(3, 4, 0)$ (6 marks)

QUESTION SIX

- a) Find the Fourier series for function of f function $f \in \mathbb{K} 2\pi$, which is given in

the interval $(-\pi, \pi)$ by
$$\begin{cases} 0, & \text{for } -\pi < -t \leq 0 \\ t & \text{for } 0 \leq t < \pi \end{cases}$$

and find the sum of the series for $t = 0$ (8 marks)

- b) The odd and periodic function f of period 2π is given in the interval

$(0, \pi)$ by $f(t) = \begin{cases} \sin t, & \text{for } t \in (0, \frac{\pi}{2}) \\ -\sin t & \text{for } t \in (\frac{\pi}{2}, \pi) \end{cases}$ (7 marks)