The Management University of Africa

Sponsored by the Kenya Institute of Management

UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 323: CALCULUS III - APPLICATIONS

DATE: 1ST AUGUST 2024

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provide**

QUESTION ONE

a) Verify Rolle's Theorem in the interval (2, 4) for the function

f(x) $= x^2$ - 6x + 8 **(5 marks)**

b) Use the sum of the first 100 terms to approximate the sum of the series

$\sum \frac{1}{(x^3+1)}$. Estimate the error involved in this approximation. **(5 marks)**

c) Form the partial differentiation equation by eliminating the constants a and b from the equation $(x + a)^2 + (y + b)^2 + z^2 = 4$ **(5 marks)**

d) Evaluate $\int_0^3 \int_1^2 (x^2 + 3y^2)\, dy\, dx$ **(5 marks)**

e) Evaluate $\int_C x^4\, dx + x\, y\, dy$, where C is the triangular curve consisting of the line segments from (0, 0) to (1, 0) , from (1, 0) to (0,1) , and from (0, 1) to (0,0)

(5 marks)

QUESTION TWO

a) Given $\int_0^3 (2\,x^2 - 1)\, dx = 15$, find C such that f(c) equals the average value of f(x) = $2x^2 - 1$ over [0, 3] **(3 marks)**

b) For the function f(x) = $x^2 + 2\,x$, find all the values of C that satisfy the Lagrange Theorem over the interval (-4, 4) **(4 marks)**

c) Determine if Rolle's Theorem applies for the function

f(x) = 3x - $3x^3$ on [- 2, 1]. If it does, find x = c guaranteed by it for f(x) = 3x - $3x^3$ on [-2, 1] **(4 marks)**

d) Use L'Hospital's Rule to evaluate $\lim_{t \to 1} \frac{5\,t^4 - 4\,t^2 - 1}{10 - t - 9\,t^3}$ **(4 marks)**

QUESTION THREE

a) Use Taylors Theorem to expand the function f(x) = $\sqrt[3]{x}$ in ascending powers of x up to degree 2. Approximate the function at a = 8

(6 marks)

b) Use Maclaurin's theorem to expand ln(1 +x) in ascending powers of x as far as the term $\ln x^5$ **(6 marks**

c) Find the sum $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ to within $\frac{1}{10}$ **(3 marks)**

QUESTION FOUR

a) Use the chain to evaluate y = $(3x+2)^5$ **(3 marks)**

b) If z = 2x^2 + 3xy + y^3, find $\frac{dz}{dx}$ and $\frac{dz}{dy}$ **(4 marks)**

c) Find the second partial derivatives of

a. f (x, y) = x^3 + x^2y^3 - 2 y^2 **(4 marks)**

d) A farmer has 100m of wire-mesh to fence a rectangular enclosure. What is the greatest area can he enclose with the wire- mesh? **(4 marks)**

QUESTION FIVE

a) Evaluate the integrals $\int_0^3 \int_1^2 x^2y\, dy\, dx$ **(3 marks)**

b) Evaluate $\int_0^1 \int_0^1 \int_0^x (x-2\,y+z)$ dz dy dx **(6 marks)**

c) Evaluate $\int_C$ y dx + z dy + x dy, where C consists of the line segment from C 1 from (2, 0, 0) to (3, 4, 5) followed by the vertical line segment and C 2 from (3, 4, 5) to (3, 4, 0) **(6 marks)**

QUESTION SIX

a) Find the Fourier series for function of f unction f $\in K\,2\pi$, which is given in the interval $(-\pi, \pi)$ by $\begin{cases} 0, for\ <-t \leq 0 \\ t\ for\ 0 \leq \pi \end{cases}$

and find the sum of the series for t = 0 **(8 marks)**

b) The odd and periodic function f of period 2 π is given in the interval

$(0, \pi)$ by f (t) = $\begin{cases} \sin t, for\ t \in (0, \frac{\pi}{2}) \\ -\sin t\ for \in (\frac{\pi}{2}, \pi) \end{cases}$ **(7 marks)**