



UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 323: CALCULUS III: APPLICATIONS

DATE: 10TH APRIL 2026

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provided.**

QUESTION ONE

- a) Verify the Mean Value Theorem for the function $f(x) = x^3$ in the interval $[-1, 3]$
(4 marks)
- b) Use Taylor's series to evaluate the function $f(x) = x^3 - 10x^2 + 6$ at $x = 3$
(6 marks)
- c) If $f(x,y) = 4 - x^2 - 2y^2$, find $f_x(1,1)$ and $f_y(1,1)$ and interpret these numbers as slopes
(4 marks)
- d) Evaluate the integrals $\int_0^3 \int_1^2 (x^2 + 3y^2) dy dx$
(4 marks)
- e) Find the absolute local minimum value and the absolute local maximum value of f given the function $f(x) = 3x^4 - 16x^3 + 18x^2$ in the interval $-1 \leq x \leq 4$
(7 marks)

QUESTION TWO

- a) Verify if the function $x^2 + 2x - 8$ satisfies Lagrange mean value theorem in the interval $[-4, 4]$
(4 marks)
- b) Determine if Rolle's Theorem applies for the function
 $f(x) = x^4 - 2x^2$ on $[-2, 2]$
(5 marks)
- c) Find the average value of the function $f(x) = \frac{x}{\sqrt{(x^2+16)}}$ over the interval $(0, 3)$
(6 marks)

QUESTION THREE

- a) Find the remainder when $P(x) = f(x) = x^6 - 3x^5 + 6$ is divided by $x + 2$
(3 marks)
- b) Evaluate the Taylor's series for the function $f(x) = e^x$ at $x = 0$
(6 marks)
- c) Find the first three terms in the Maclaurin's series for $\sin^2 x$ at $x = 0$
(6 marks)

QUESTION FOUR

- a) Find the critical numbers of the function $3t^4 + 4t^3 - 6t^2$ **(4 marks)**
- b) Use the L'Hospital rule to evaluate $\lim_{t \rightarrow 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$ **(3 marks)**
- c) Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection, local maxima and minima. Use this information to sketch the curve. **(8 marks)**

QUESTION FIVE

- a) Evaluate the integrals $\int_0^2 \int_0^1 (2y^2x^2 + 3) dy dx$ **(4 marks)**
- b) Evaluate $\int_2^3 \int_{-1}^4 \int_1^0 (4x^2y - z^3) dx dy dz$ **(6 marks)**
- c) Use triple integral to determine the volume of the region $Z = 4 - xy$ and above the region in the xy -plane defined by $0 \leq x \leq 2$, $0 \leq y \leq 1$ **(5 marks)**

QUESTION SIX

- a) For a periodic function of period 2π , prove that
$$\int_{\sigma}^{\beta} f dx = \int_{\sigma+2\pi}^{\beta+2\pi} f dx$$
 (4 marks)
- b) Expand the periodic function $f = x^2$, $0 \leq x \leq l$ of period l , in the a series of cosines only. Also find the sum of the series at $x = 0, l$. **(11 marks)**