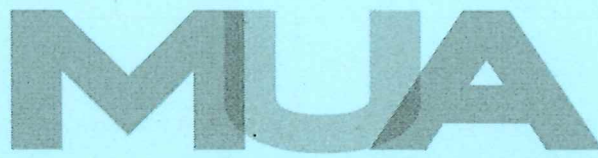


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The
Management
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UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 443: NUMERICAL ANALYSIS

DATE: 4TH DECEMBER 2024

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. Write all your answers in the Examination answer booklet provided.

QUESTION ONE

a) Find the relative error in the quotient when $0.3614 \div 0.3613$ (3 marks)

b) Use synthetic division method to evaluate the function

$$P(x) = 2x^4 - 3x^2 + 3x - 4 \text{ when } r = -2 \quad (4 \text{ marks})$$

c) Use the finite difference table to detect and correct one error in the table below

X	0	1	2	3	4	5	6	7	8	9	10
f(x)	3	2	3	6	11	18	21	38	51	66	83

(6 marks)

d) Evaluate the integral $\int_0^{1.2} e^x dx$, taking $n = 6$ using Simpson's rule

(5 marks)

e) Obtain the first and second derivatives of the function tabulated below at points $x = 1.1$ (7 marks)

X	1	1.2	1.4	1.6	1.8	2.0
Y	0	0.128	0.544	1.298	2.440	4.02

QUESTION TWO

a) Suppose $x = \frac{5}{7}$, $y = \frac{1}{3}$ and that five digit chopping is used for arithmetic calculations involving x and y . Find the absolute error and relative error involved in $x + y$ (3 marks)

b) Convert $P(x) = -8x^3 + 7x - 8x^4 + 2x^5 - x^2 + 3$ to nested form and evaluate $P(5)$ (6 marks)

c) Apply Lagrange's interpolation formula to find a polynomial which passes through the points $(0, -20)$, $(1, -12)$, $(3, -20)$ and $(4, -24)$ (6 marks)

QUESTION THREE

a) Use the table below:

X	1	1.1	1.2	1.3	1.4	1.5
Y	5.4680	5.6665	5.9264	6.2551	6.6601	7.1488

to find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $x = 1.0$ (7marks)

b) Compute the values of $f'(3.1)$ and $f''(3.1)$ using the following table

X	1	2	3	4	5
f(x)	0	1.4	3.3	5.6	8.1

(8 marks)

QUESTION FOUR

a) Use the trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2 dx$ and find the error involved (9 marks)

b) Suppose it is found experimentally that two physical variables x and y are related as shown in the table below

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	3.1	4.0	4.2	3.8	2.9	2.8	2.7

Find the difference between the error involved by use of trapezoidal rule and Simpson's rule in the integral of a function described by this tabular data.

(6 marks)

QUESTION FIVE

a) Construct the forward finite difference table and the horizontal finite difference table for $y = f(x) = x^3 + 2x + 1$ for $x = 1, 2, 3, 4, 5$. (7 marks)

b) Find and correct a misprint in the table below:

$Y_i = 1, 3, 11, 31, 69, 113, 233, 351, 521, 739, 1011$

(8 marks)

QUESTION SIX

- a) Convert $P(x) = 6x^2 - 7x + 3x^4 + 11 - 2x^3$ to nested form and evaluate $P(3)$
(5 marks)

- b) Find the polynomial of degree three which takes the four values listed in the table below

x_k	0	1	2	3
y_k	0	1	16	8

(5 marks)

- c) Use synthetic division to evaluate

$$\frac{4x^3 - 8x^2 - x + 5}{2x - 1}$$

(5 marks)

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Tables and Formulae

- 1) XE = exact or true value of quantity, XA = approximate value, Absolute error = $|XE - XA|$, Relative error = $\left| \frac{XE - XA}{XE} \right|$
- 2) A polynomial of degree n has the form, $P_n(x) = ax^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, $a_n \neq 0$
 - a. Taylor's Theorem, $f(a) = f(a+h) = f(a) + f'(a)h + \frac{f''(a)h^2}{2!} + \frac{f'''(a)h^3}{3!} + \frac{f^{(4)}(a)h^4}{4!} + \dots$
 - b. Maclaurin's Theorem, $f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{(4)}(0)x^4}{4!} + \dots$
- 3) Lagrange interpolation, $L_0y_0 + L_1y_1 + L_2y_2 + L_3y_3 + \dots + L_ny_n$, where

$$L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots(x_0-x_n)}$$

$$L_1 = \frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)}$$

$$L_2 = \frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)}$$

$$L_3 = \frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_n)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)\dots(x_3-x_n)}, \dots$$
- 4) Newton's forward difference formula, $y = y_0 + \frac{u\Delta y}{1!} + \frac{u(u-1)\Delta^2 y}{2!} + \frac{u(u-1)(u-2)\Delta^3 y}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 y}{4!} + \dots$, where $u = \frac{x-x_0}{h}$
- 5) Interpolating polynomial formula, $P_n(x) = L_0 = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots(x_0-x_n)}$
- 6) Trapezoidal rule, $\int_a^b f(x)dx = \frac{h}{2} [(sum\ of\ ends) + 2(sum\ of\ middles)]$, where $h = \frac{b-a}{n}$, n = number of stripes
- 7) Simpson's rule $\int_a^b f(x)dx = \frac{h}{3} [(sum\ of\ ends) + 4(sum\ of\ odd\ stripes) + 2(sum\ of\ even\ stripes)]$, where $h = \frac{b-a}{n}$, n = number of stripes, Also n = number of ordinates - 1
 Simpsons maximum possible error = $\frac{(b-a)h^4}{180} m$, where m is the maximum $f^{(4)}$ on $[a,b]$ [ie 4th derivative]

